Calculating the Risk of Structural Failure

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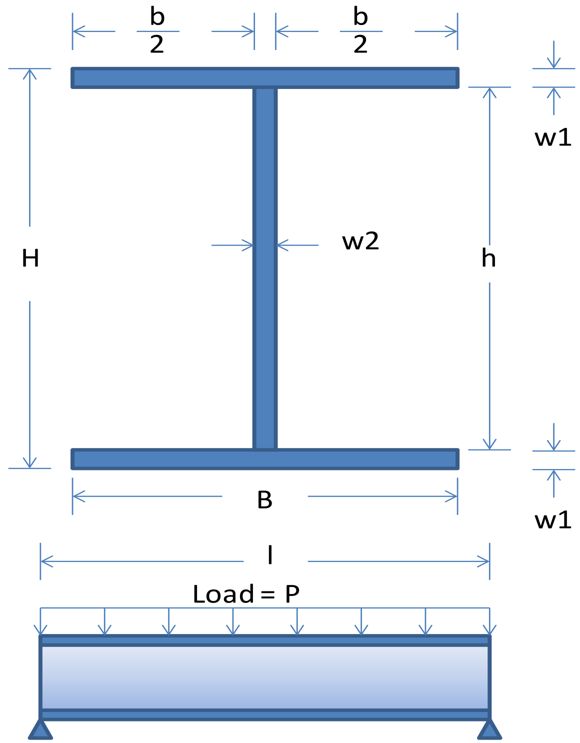
Bob Graber
STARGroup Solutions, LLC
robert.graber@stargroup.solutions

Designing a Structure for Reliability

- Two processes or methods can be used to design a structure to withstand its intended environment without failure
  - Process 1. A Factor of Safety (FOS) method is used to determine the allowable stresses and an appropriate design configuration
  - Process 2. Probabilistic design (stress-strength interference) methods are used to design the structure to meet a “reliability requirement” value
- We will investigate the failure “risk” implications with the two design processes

I-Beam Example for Discussion

- A distributed estimated mean load of 2300 lb/in will be incurred on a surface that will be resisted by an envisioned I-beam of length L where the I-beam rests on supports at its ends
- A <u>preliminary design</u> (based perhaps on previous designs) is shown with the dimensions shown



Let:

Material: 7075 – T73 Al

w1 = 0.25 in.
w2 = 0.25 in.
B = 3.00 in.
h = 3.50 in.
l = 25.00 in.
P = 2300 lbs/in (nominal)
and
H = h + 2*w1
b = B – w2

Finding How Much Stress is within a Structure

- We may use standard mechanical engineering concepts to determine the stresses in common structural components given the loads they bear.
- For instance, the properties of beams include the bending moment or stress moment (in say, pound*inches), M, and is defined to be

$$M = \frac{S * I}{c}$$

where S is the elastic unit stress at outer fiber whose distance from the neutral axis is c; and I is the rectangular moment of inertia of the cross section. This formula for M is also defined as **strength of beams** formula. The term **section modulus** is given to the value of I/c.

Finding How Much Stress is within a Structure (cont.)

- The bending moment, M, of the I-beam is also a function of the distributed load, P, and the length, L and is defined to be

$$M_x = \frac{P * x}{2}(l - x)$$

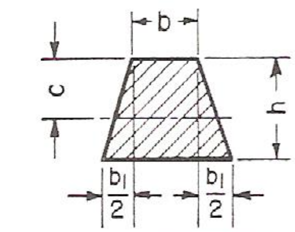
  where x is the distance from an end support.
- The maximum bending moment is where x = ½ L, so

$$M_{max} = \frac{P * L^2}{8}$$

- We can equate M_{max} to the bending or stress moment, M, from the equation on the previous page

Beam Section Modulus (I/c) Formulas

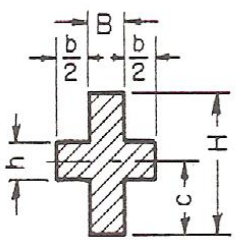
From Marks’ Standard Handbook for Mechanical Engineers, Eighth Edition, p 5-31.

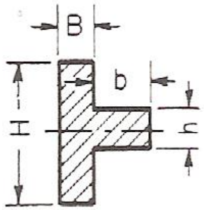


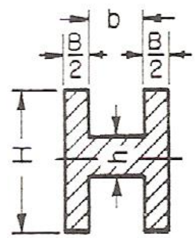
$$I = \frac{6b^2 + 6bb_1 + b_1^2}{36(2b + b_1)} h^3$$

$$c = \frac{1}{3}\,\frac{3b + 2b_1}{2b + b_1} h$$

$$\frac{I}{c} = \frac{6b^2 + 6bb_1 + b_1^2}{12(3b + 2b_1)} h^2$$

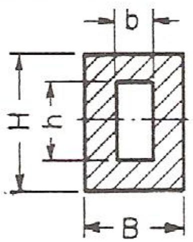


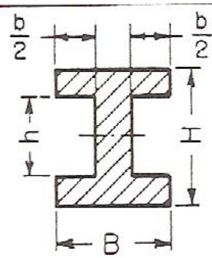


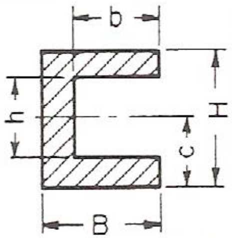


$$I = \frac{BH^3 + bh^3}{12}$$

$$\frac{I}{c} = \frac{BH^3 + bh^3}{6H}$$







$$I = \frac{BH^3 - bh^3}{12}$$

$$\frac{I}{c} = \frac{BH^3 - bh^3}{6H}$$

Finding How Much Stress is within a Structure (cont.)

- The properties of various beam cross sections defined in mechanical engineering handbooks give the equation for the section modulus for the I-beam:

$$\frac{I}{c} = \frac{BH^3 - bh^3}{6H}$$

- Including the flange thicknesses through substitution:

$$\frac{I}{c} = \frac{B * (h + 2w1)^3 - (B - w2) * h^3}{6(h + 2w1)}$$

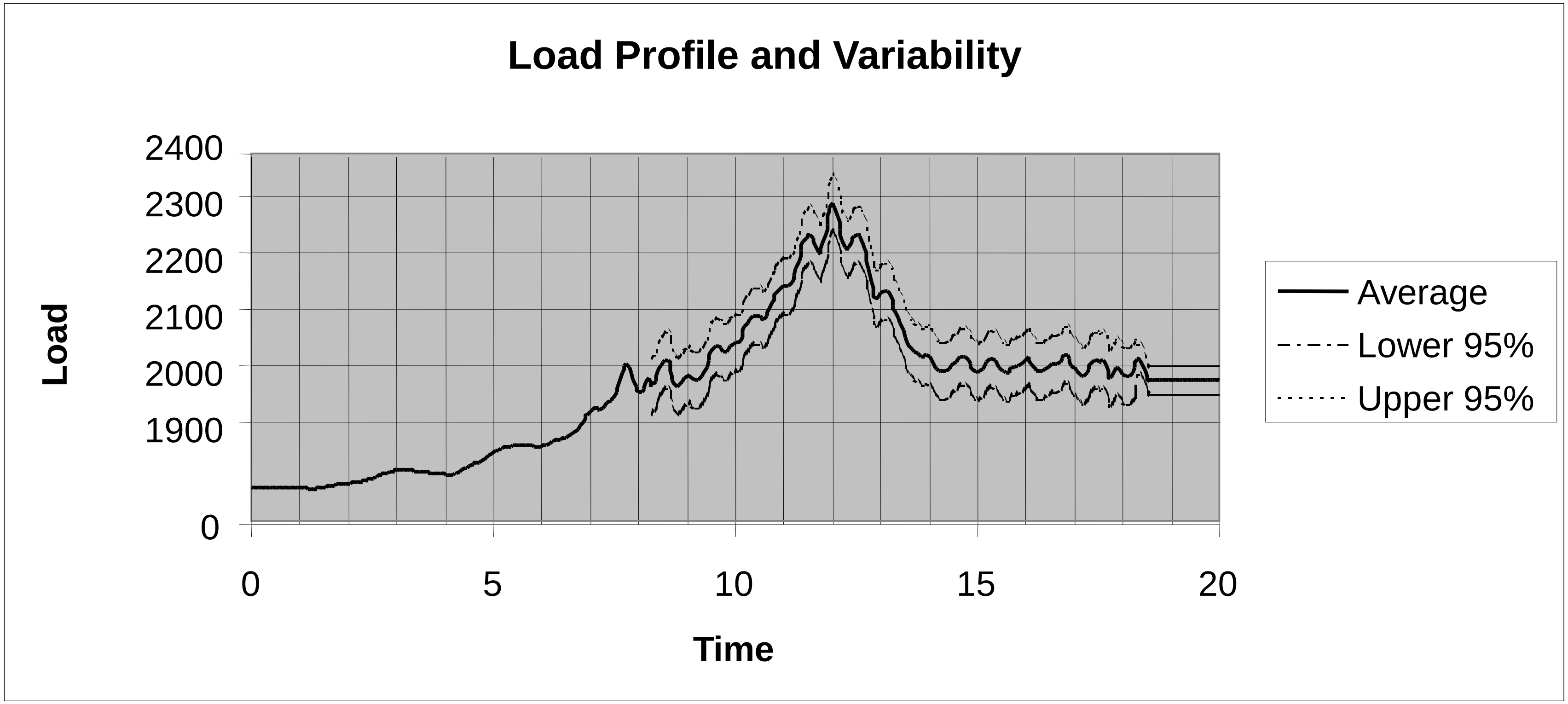
- From these equations, we now have an equation for the **failure governing stress** within the I-beam in terms of the dimensions of the beam and the load:

$$S = \left[\frac{\frac{P * l^2}{8}}{\frac{B * (h + 2w1)^3 - (B - w2) * h^3}{6(h + 2w1)}} \right]$$

- For example, if we know the **distributed load P to be, say, exactly 2300 lbs** we can calculate the **failure governing stress, S, to be 58,203 psi or ~ 58 ksi** with all dimensions at their nominal values.

Generating Load Values from Test Data

- An example measured load profile was developed from an average of several observed load profiles centered around their peak loads. The mean load is estimated to be 2300 pounds, with the maximum (95%) bound to be 2360 pounds.



Factor of Safety Design Method

- In a classic structural design process a permissible maximum stress is established and used in the design. This is the allowable stress, or the **design stress.**
- The design stress is determined by dividing the applicable material property – **yield strength**, **ultimate strength**, or **fatigue strength** – by a **factor of safety.**

$$\text{Design Stress} = \frac{\text{Design Strength}}{\text{Factor of Safety (FOS)}}$$

(As calculated or as determined from basis values)

$$\text{Design Strength} = \text{Design Stress} \times \text{Factor of Safety (FOS)}$$

- A FOS is usually prescribed by historical precedence, and we must determine the design strength and/or stress

Finding or Estimating Design Strength Data with the Factor of Safety Method

- Strengths of various materials may be obtained from handbooks like Metallic Material Properties Development and Standardization Handbook (MMPDS-03) that originated from the old MIL-HDBK-5J
  - For a given material, the A-basis and B-basis strength may be provided
  - A-basis value: At least 99 percent of the population of values is expected to equal or exceed this tolerance bound with a confidence of 95 percent.
  - B-basis value: At least 90 percent of the population of values is expected to equal or exceed this tolerance bound with a confidence of 95 percent.

- Also, given an A-basis and B-basis value, if we treat the basis values as random variables following a normal distribution, the mean and standard deviation values of the strength may be calculated
  - For A-basis, A-basis value = $\mu_S + 2.688 * \sigma_S$
  - For B-basis, B-basis value = $\mu_S + 1.786 * \sigma_S$
  - Then μ_S = (2.98*B-basis) – (1.98*A-basis),
  - σ_S = (B-basis) - (A-basis) / .902

Factor of Safety Method – Designing Strength to Stress

- For example, from MIL-HDBK-5J, the material properties of 7075 T73 Aluminum has an A-basis for yield strength of 60 ksi (thousand pounds per square inch), and the B-basis tensile yield strength is 63 ksi.
  - Note that the calculated mean yield strength is then 70 ksi, with a standard deviation is approximately 3.33 ksi.
- Values for the factor of safety are usually program specific
  - Typical FOS are between 1.5 and 4.0. However, the minimum factor of safety for metallic flight structures stated in a NASA Structural Design and Verification Document is 1.25 for yield and 1.4 for ultimate in Protoflight articles.
- For this material then, the **allowable stress** with a FOS of 1.25 would be **60/1.25 = 48 ksi for yield**
  - That is, 99% of the material used will not yield at a stress of 48 ksi with a 95% confidence
- A structure design that is made of 7075 T73 Aluminum should, therefore, be able to withstand a stress level of 48 ksi without yielding

Factor of Safety Method – Designing Strength to Stress (cont.)

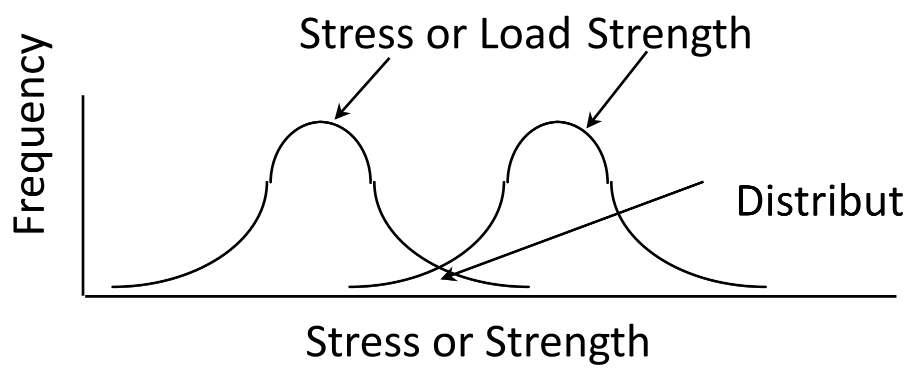
- In our example then, **if the distributed load is (exactly) 2300 lb/in**, the resultant I-beam configuration stress of about 58 ksi is quite a bit higher than the allowable, and **the design would be inadequate** taking in account the required 1.25 factor of safety.
- In order to meet the allowable stress, the design must be altered. One logical solution to this problem would be to increase the flange width dimensions w1 and w2. Using the equations above, in order **to meet a 48 ksi yield allowable, w1 and w2 would need to be increased to about 0.30 inches** each.
- However, it is common to use the <u>maximum</u> load case in the determination of the design stress for comparison to the allowable stress. In our example case, **if a maximum load of 2360 is estimated**, the failure governing stress level becomes about 60 ksi, **the flange widths would need to be increased to .31 inches to meet the allowable 48 ksi yield stress level.**

The Stress-Strength Interference Method

Stress-strength interference (sometimes called load-strength interference) is a statistical analysis of the loads or stresses on an item in comparison to the strength of the item. When there are variations in the material strength and applied stress, their statistical distributions can be measured or estimated.

The terms, “Stress” or “Load”, might refer to a mechanical stress or strain, a voltage, or internally generated stresses such as temperature.

The term, “ Strength”, might refer to any resisting physical property, such as hardness, strength, melting point, or adhesion.



Design using Stress-Strength Interference

In the design of structural elements , we begin with an assumption that the strength and applied stress for a structure can be represented as two independent random variables. In many cases, the variables may be described using a normal probability density distribution with a mean and variance.

	Mean	Variance
Strength	μ_S	σ_S^2
Stress or Load	μ_L	σ_L^2

Then in a load application case, the probability that the strength is greater than the stress or load (the interference) can be expressed as:

$$R = \Phi\left(\frac{\mu_S - \mu_L}{\sqrt{\sigma_L^2 + \sigma_S^2}} \right)$$

Where R = the **reliability or probability of not failing**, and

Φ = the standard cumulative variate of the normal distribution

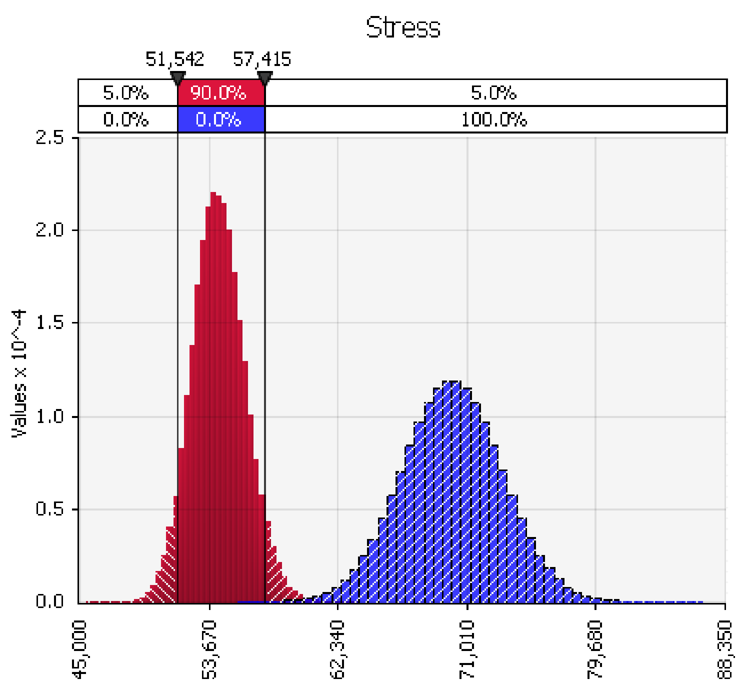
Design using Stress-Strength Interference (cont.)

- If we know the means and variances of the stress and strength, we may select a design reliability goal and vary the stress-strength parameters to meet that goal
- In our example, we will select a goal reliability (say .99998 or 1 chance of failure in 50,000) and vary the relevant flange thickness to meet that goal
- The values for the parameters used in our example are shown here:

Measure/Parameter	Mean Value	Standard Deviation
w1	0.25 in	0.01 in
w2	0.25 in	0.01 in
B	3.00 in	0.01 in
h	3.50 in	0.01 in
l	25.00 in	0.03 in
P	2300 lb/in	20 lb/in
Strength (yield)	70 ksi	3.33 ksi

Calculating the Reliability/Probability of Failure

- We can use the stress-strength equation for evaluation
- We may also use a simulation like @Risk for evaluation



Statistics

	Stress	Strength
Cell	Sheet1!I15	Sheet1!I18
Minimum	45,424.28	55,682.09
Maximum	63,698.59	86,777.49
Mean	54,388.85	70,000.02
Mode	54,358.04	70,041.74
Median	54,335.92	69,999.94
Std Dev	1,788.75	3,330.10
Skewness	0.1713	0.0004
Kurtosis	3.0597	3.0018
Values	100000	100000
Errors	0	0
Filtered	0	0
Left X	51,542	51,542
Left P	5.0%	0.0%
Right X	57,415	57,415
Right P	95.0%	0.0%
Dif. X	5,873.55	5,873.55
Dif. P	90.0%	0.0%
1%	50,445.76	62,252.77
5%	51,541.87	64,522.52
10%	52,135.60	65,732.37

Comparing Design Options Using Stress-Strength Calculations

- Using the interference equation, the reliability of the modified design using the FOS method is calculated to be 0.99999999903 with an associated probability of failure of 9.7 E-10 or about 1 in a billion.
- Suppose we would be willing to accept a risk of 1 in 50,000 of an individual I-beam failure (corresponding to a reliability of .99998.)
  - Using the stress-strength relationships and methodology shown in the previous paragraphs, many options for I-beam design configurations may be used to converge on a design that meets the reliability goal.
  - In our case, while using the max load value of 2360 lb/in, and using only the flange width as the adjustment variable, a flange width of .275 inches would result in a mean stress level of about 57.4 ksi and a calculated reliability level of .999983.

Weight/Mass Considerations

- In our original proposed example:
  - The cross section area of the I-beam is 2.375 in^2 and the total volume of aluminum is 59.375 in^3.
  - The density of the 7075 aluminum is .101 lbs/in^3, so the total weight of the original I-beam is 5.997 lbs.
  - With the altered design that meets the 48 ksi allowable, the total weight of the I-beam is 7.436 lbs
- We want to consider using the stress-strength design method to see the differences

Comparison of Design Process Methods

- A summary of reliability calculations shows that reducing the requirement of reliability to about .99998 from the FOS method that results in 9 nines can result in a savings of over 11% in weight.

Design Process	Estimated Failure Probability	Resulting Weight	Weight Reduction Percent
Use of 1.25 Factor of Safety on Yield Strength	$\sim 9.7 \times 10^{-10}$	7.436 lbs	-
Design to Reliability of .99998 at max load	~1 in 50,000	6.597 lbs	11.3%
Design to Reliability of .99998 at mean load	~1 in 50,000	6.429 lbs	13.5%

Summary

- Of course, many alternative design options could be evaluated that involve other structural dimensions, variations, shapes, materials, etc.
- However, the methodology of determining a failure governing stress level as a function of the structure parameters and loads, etc. (along with their variability and/or uncertainties) allows for quick design trades and application of probabilistic methods.